Introduction to Analysis in One Variable

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This is a text for students who have had a three course calculus sequence, and who are ready to explore the logical structure of this area of mathematics, which forms the backbone of analysis. This is intended for a one-semester course. An accompanying text, *Introduction to Analysis in Several Variables*, can be used in the second semester of a one year sequence.

The initial part of our development is the system of numbers, starting with the natural numbers, then the integers and the rational numbers, and then the system of real numbers, as ideal limits of sequences of rational numbers.

This solid development of the real number system forms a foundation for our further development. We proceed to further spaces, both higher-dimensional Euclidean spaces $\mathbb{R}^n$ and general metric spaces. Then we study functions on such spaces, with an emphasis on sequences of such functions, and on series (taking values in $\mathbb{R}^n$), with a particular emphasis on power series. Such background paves the way for a presentation of the major topics in one-variable calculus, in Chapter 4.

We also bring in complex numbers, which we see as a very important ingredient in the understanding of basic analysis. In particular, we develop the theory of the exponential function, not only for real arguments, but also for complex arguments, and show how such a study leads to a self-contained presentation of the trigonometric functions, unified with the exponential function. This in turn leads to a simplification of the study of Fourier series, as is seen in the treatment of that subject.

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