Linear Algebra

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Linear algebra is an important gateway connecting elementary mathematics to more advanced subjects, such as multivariable calculus, systems of differential equations, differential geometry, and group representations. The purpose of this work is to provide a treatment of this subject in sufficient depth to prepare the reader to tackle such further material.

We start with vector spaces over the set $\mathbb{R}$ of real numbers or the set $\mathbb{C}$ of complex numbers, and linear transformations between such vector spaces. We treat these two cases simultaneously, and use the label $\mathbb{F}$ to apply to either $\mathbb{R}$ or $\mathbb{C}$.

Later on we consider vector spaces over general *fields*, denoted $\mathbb{F}$, and the reader can appreciate the early material on this more general level with minimal effort. Going further, we extend the theory of vector spaces over a field to that of modules over a ring, and consider new phenomena that arise in this expanded setting.

Features of our development include a clean treatment of determinants, based on three simple rules as a definition, rather than on a relatively inscrutable defining formula. We also emphasize contact between linear algebra and geometry and analysis, including such topics as spectral theory of self-adjoint, skew-adjoint, orthogonal, and unitary transformations, and the matrix exponential.

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