How Smooth is a C^2 Surface?

Michael Taylor

Let $M \subset \mathbb{R}^n$ be a k-dimensional submanifold, smooth of class C^2, i.e., locally the graph of C^2 maps $V \to V^\perp$, $V \subset \mathbb{R}^n$ a linear space of dimension k. We ask the question

how smooth is M?

This question might appear to be trivial, but actually it's not. In our talk we analyze the question, bringing in the notions of harmonic coordinates, the Riemann tensor, casting the Ricci curvature equation in harmonic coordinates, elliptic regularity, L^p-Sobolev spaces, and bmo.

We draw conclusions about the geodesic flow on M, and, going further, about microlocal propagation of singularities for the wave equation $\partial_t^2 u - \Delta u = f$ on $\mathbb{R} \times M$, and on the decay of solutions to damped wave equations

$$\partial_t^2 u + a(x)\partial_t u - \Delta u = 0,$$

under control conditions (seen to be relevant due to results on the geodesic flow hinted above). Results here would be inaccessible from a naive answer to the initial question posed above.

This material was developed in the following works.

1. M. Taylor, Riemannian manifolds with bounded Ricci tensor, Chapter 3, §10 of *Tools for PDE*, AMS, 2000.

2. M. Taylor, Propagation of singularities, Chapter 3, §11 of *Tools for PDE*, AMS, 2000.

3. M. Taylor, Remarks on nonsmooth Riemannian manifolds, Notes, available at https://mtaylor.web.unc.edu/notes (item #5, elliptic equations)

4. M. Anderson, A. Katsuda, Y. Kurylev, M. Lassas, and M. Taylor, Boundary regularity for the Ricci equation, geometric convergence, and Gel'fand's inverse boundary problem, Invent. Math. 158 (2004), 261–321.

5. M. Taylor, Existence and regularity of isometries, Trans. AMS 358 (2006), 2415–2423.

6. M. Taylor, Anderson-Cheeger limits of smooth Riemannian manifolds, and other Gromov-Hausdorff limits, J. Geom. Anal. 17 (2007), 365–374.

7. M. Taylor, Wave decay on manifolds with bounded Ricci tensor and related estimates, J. Geom. Anal. 25 (2015), 1018–1044. (Building on Rauch-Taylor, 1974)