Introduction to Analysis in Several Variables
(Advanced Calculus)

Michael Taylor

This text was produced for the second part of a two-part sequence on advanced calculus, whose aim is to provide a firm logical foundation for analysis, for students who have had 3 semesters of calculus and a course in linear algebra. The first part treats analysis in one variable, and the text *Introduction to Analysis in One Variable* was written to cover that material. The text at hand treats analysis in several variables. These two texts can be used as companions, but they are written so that they can be used independently, if desired.

An introductory chapter treats background for multivariable calculus. This includes sections on one-variable calculus, on n-dimensional Euclidean space $\mathbb{R}^n$, on vector spaces and linear transformations, and on determinants. The text proceeds to material on analysis in several variables, from differential calculus to the n-dimensional integral, to calculus on n-dimensional surfaces in $\mathbb{R}^k$, and generalizations known as manifolds.

Topics covered include systems of differential equations, and their relation to vector fields, differential forms, as a coordinate-independent way to set up integrals, with numerous applications to such topics as holomorphic and harmonic functions, and to topological implications, involving "degree theory." We also study the geometry of surfaces, from geodesics to curvature, relating the latter to degree theory via Gauss-Bonnet theorems. A chapter on Fourier analysis treats both n-dimensional Fourier series, the Fourier transform on $\mathbb{R}^n$, and the theory of spherical harmonics on n-dimensional spheres, as well as Fourier analysis on matrix groups. A final topic in the appendix previews an extension of degree theory, known as de Rham theory. Such topics spotlight the unity of the various analytical and geometric aspects of Advanced Calculus.

Contents

6. Further results on harmonic functions
7. Beyond degree theory – introduction to de Rham theory