Introduction to Complex Analysis

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This text is designed for a first course in complex analysis, for beginning graduate students, or well prepared undergraduates, whose background includes multivariable calculus, linear algebra, and advanced calculus. In this course the student will learn that all the basic functions that arise in calculus, first derived as functions of a real variable, such as powers and fractional powers, exponentials and logs, trigonometric functions and their inverses, and also many new functions that the student will meet, are naturally defined for complex arguments. Furthermore, this expanded setting reveals a much richer understanding of these functions.

The central objects in complex analysis are functions that are complex-differentiable (i.e., holomorphic). One early goal is to establish an equivalence between being holomorphic and having a convergent power series expansion. Half of this is done in Chapter 1, the other half in Chapter 2, after the introduction of the Cauchy integral theorem as a major theoretical tool. Other applications of this essential tool occupy much of the rest of the text.

We emphasize the connection of complex analysis to other facets of analysis and geometry. As some distinguishing features of the treatment here, we mention the following.
(1) An emphasis on Fourier analysis, both as an application of basic results and as a tool of more general applicability in analysis.
(2) Use of geometrical techniques, clarifying the study of holomorphic functions as conformal maps, and extending the usual study to more general surfaces.
(3) Connections with differential equations. Techniques of complex analysis are brought to bear, from the classical case of Bessel equations to more general situations. Connections are made to how some of these important differential equations arise from a study of wave equations and diffusion equations on various domains in higher dimensional Euclidean space.

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