Multivariable Calculus

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This is a text for students with a background in one-variable calculus, who are ready to tackle calculus in several variables. It is designed for the honors section of the third-semester calculus course at the University of North Carolina.

Chapter 1 presents a brisk review of the basics of calculus in one variable: definitions and elementary properties of the derivative and integral, the fundamental theorem of calculus, and power series. One might skim over this introductory chapter to see if a refresher is needed for some of this material.

Multivariable calculus is done on multidimensional spaces. Chapter 2 introduces algebraic tools useful for this study, involving a look at n-dimensional Euclidean space $\mathbb{R}^n$, more general vector spaces, linear transformations, matrices, and determinants, and a study of the cross product on $\mathbb{R}^3$. We proceed to a study of curves in Euclidean space in Chapter 3. Material presented here on arclength of curves leads to a unified treatment of exponential and trigonometric functions.

Chapter 4 treats the derivative of a function of several variables, including higher derivatives and multivariable power series, and the inverse and implicit function theorems. Chapter 5 develops the integral of functions of several variables. A key result here is the formula for change of variables of multiple integrals, which makes essential use of concepts from Chapter 4. Chapter 6 studies smooth surfaces in Euclidean space, and differential and integral calculus on such surfaces. It also discusses the concept of a manifold, as a generalization of the notion of a surface.

Appendices provide supplementary material, from foundational results on the real number system, to basic results on sequences and series of functions, to auxiliary results on linear algebra, intended to interest the ambitious reader.

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